

3-1

Lines and Angles

Common Core State Standards

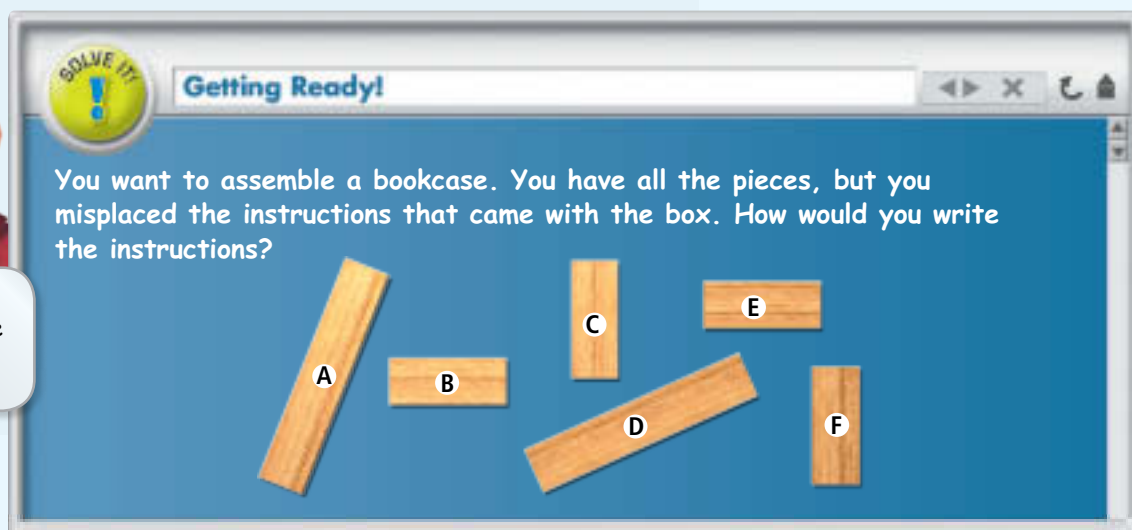
G-CO.A.1 Know precise definitions of . . . parallel line.
Prepares for G-CO.C.9 Prove theorems about lines and angles.

MP 1, MP 3, MP 6

Objectives To identify relationships between figures in space
 To identify angles formed by two lines and a transversal



Try visualizing how the bookcase looks in two dimensions.



Lesson Vocabulary

- parallel lines
- skew lines
- parallel planes
- transversal
- alternate interior angles
- same-side interior angles
- corresponding angles
- alternate exterior angles

In the Solve It, you used relationships among planes in space to write the instructions. In Chapter 1, you learned about intersecting lines and planes. In this lesson, you will explore relationships of nonintersecting lines and planes.

Essential Understanding Not all lines and not all planes intersect.

take note

Key Concept Parallel and Skew

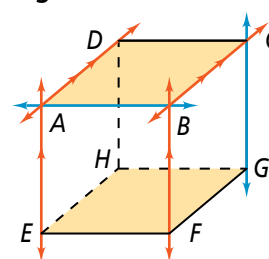
Definition

Parallel lines are coplanar lines that do not intersect. The symbol \parallel means “is parallel to.”

Symbols

$\overleftrightarrow{AE} \parallel \overleftrightarrow{BF}$
 $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$

Diagram



Use arrows to show $\overleftrightarrow{AE} \parallel \overleftrightarrow{BF}$ and $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$.

Skew lines

are noncoplanar; they are not parallel and do not intersect.

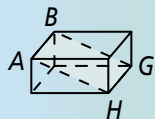
\overleftrightarrow{AB} and \overleftrightarrow{CG} are skew.

Parallel planes are planes that do not intersect.

plane $ABCD \parallel$ plane $EFGH$

Think

Parallel lines are coplanar. Which planes contain \overline{AB} ? Planes $ABCD$, $ABFE$, and $ABGH$ contain \overline{AB} . You need to visualize plane $ABGH$.



Problem 1 Identifying Nonintersecting Lines and Planes

In the figure, assume that lines and planes that appear to be parallel are parallel.

A Which segments are parallel to \overline{AB} ?

\overline{EF} , \overline{DC} , and \overline{HG}

B Which segments are skew to \overline{CD} ?

\overline{BF} , \overline{AE} , \overline{EH} , and \overline{FG}

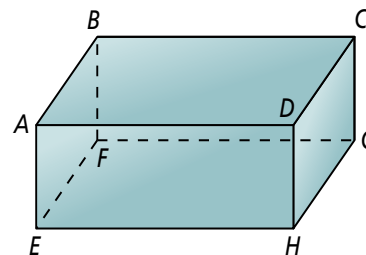
C What are two pairs of parallel planes?

plane $ABCD \parallel$ plane $EFGH$

plane $DCG \parallel$ plane ABF

D What are two segments parallel to plane $BCGF$?

\overline{AD} and \overline{DH}



Got It? 1. Use the figure in Problem 1.

a. Which segments are parallel to \overline{AD} ?

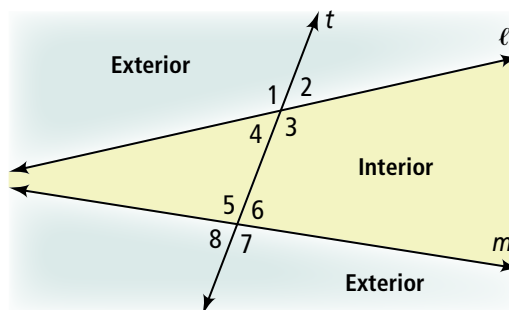
b. **Reasoning** Explain why \overline{FE} and \overline{CD} are *not* skew.

c. What is another pair of parallel planes?

d. What are two segments parallel to plane $DCGH$?

Essential Understanding When a line intersects two or more lines, the angles formed at the intersection points create special angle pairs.

A **transversal** is a line that intersects two or more coplanar lines at distinct points. The diagram below shows the eight angles formed by a transversal t and two lines ℓ and m .



Notice that angles 3, 4, 5, and 6 lie between ℓ and m . They are *interior* angles. Angles 1, 2, 7, and 8 lie outside of ℓ and m . They are *exterior* angles.

Pairs of the eight angles have special names as suggested by their positions.



Key Concept Angle Pairs Formed by Transversals

Definition

Alternate interior angles are nonadjacent interior angles that lie on opposite sides of the transversal.

Same-side interior angles are interior angles that lie on the same side of the transversal.

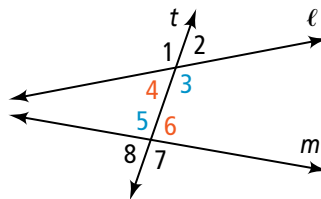
Corresponding angles

lie on the same side of the transversal t and in corresponding positions.

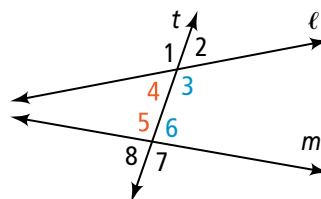
Alternate exterior angles are nonadjacent exterior angles that lie on opposite sides of the transversal.

Example

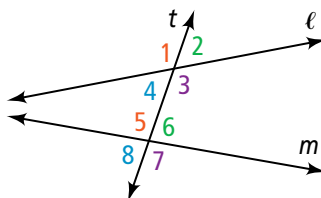
$\angle 4$ and $\angle 6$
 $\angle 3$ and $\angle 5$



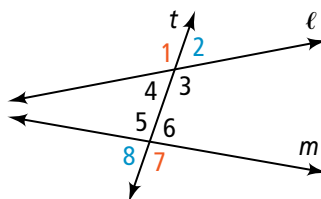
$\angle 4$ and $\angle 5$
 $\angle 3$ and $\angle 6$



$\angle 1$ and $\angle 5$
 $\angle 4$ and $\angle 8$
 $\angle 2$ and $\angle 6$
 $\angle 3$ and $\angle 7$



$\angle 1$ and $\angle 7$
 $\angle 2$ and $\angle 8$



Problem 2 Identifying an Angle Pair

Multiple Choice Which is a pair of alternate interior angles?

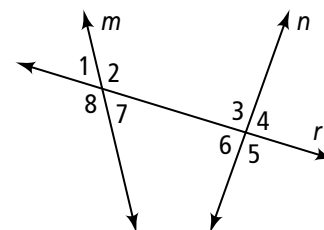
(A) $\angle 1$ and $\angle 3$

(C) $\angle 2$ and $\angle 6$

(B) $\angle 6$ and $\angle 7$

(D) $\angle 4$ and $\angle 8$

$\angle 2$ and $\angle 6$ are alternate interior angles because they lie on opposite sides of the transversal r and in between m and n .
The correct answer is C.



Think

Which choices can you eliminate?

You need a pair of interior angles. $\angle 1$, $\angle 4$, and $\angle 8$ are exterior angles. You can eliminate choices A and D.



Got It? 2. Use the figure in Problem 2. What are three pairs of corresponding angles?

Problem 3 Classifying an Angle Pair STEM

Architecture The photo below shows the Royal Ontario Museum in Toronto, Canada. Are angles 2 and 4 *alternate interior angles*, *same-side interior angles*, *corresponding angles*, or *alternate exterior angles*?

Think

How do the positions of $\angle 2$ and $\angle 4$ compare?

$\angle 2$ and $\angle 4$ are both interior angles and they lie on opposite sides of a line.



Angles 2 and 4 are alternate interior angles.



Got It? 3. In Problem 3, are angles 1 and 3 *alternate interior angles*, *same-side interior angles*, *corresponding angles*, or *alternate exterior angles*?

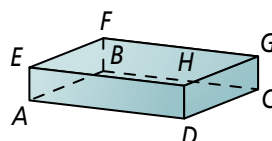


Lesson Check

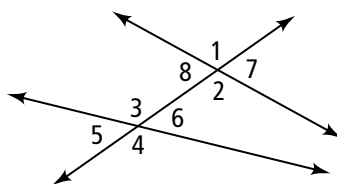
Do you know HOW?

Name one pair each of the segments, planes, or angles. Lines and planes that appear to be parallel are parallel.

1. parallel segments
2. skew segments
3. parallel planes
4. alternate interior
5. same-side interior
6. corresponding
7. alternate exterior



Exercises 1–3



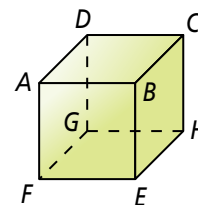
Exercises 4–7

Do you UNDERSTAND?



MATHEMATICAL PRACTICES

8. **Vocabulary** Why is the word *coplanar* included in the definition for parallel lines?
9. **Vocabulary** How does the phrase *alternate interior angles* describe the positions of the two angles?
10. **Error Analysis** In the figure at the right, lines and planes that appear to be parallel are parallel. Carly says $\overline{AB} \parallel \overline{HG}$. Juan says \overline{AB} and \overline{HG} are skew. Who is correct? Explain.

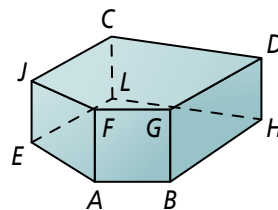


A Practice

Use the diagram to name each of the following. Assume that lines and planes that appear to be parallel are parallel.

See Problem 1.

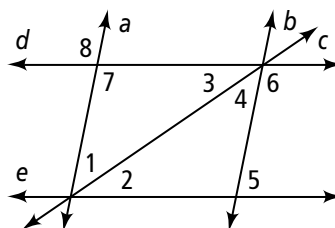
11. a pair of parallel planes
12. all lines that are parallel to \overleftrightarrow{AB}
13. all lines that are parallel to \overleftrightarrow{DH}
14. two lines that are skew to \overleftrightarrow{EJ}
15. all lines that are parallel to plane $JFAE$
16. a plane parallel to \overleftrightarrow{LH}



Identify all pairs of each type of angles in the diagram. Name the two lines and the transversal that form each pair.

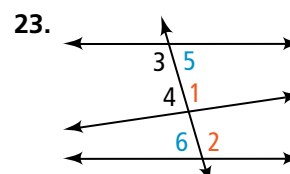
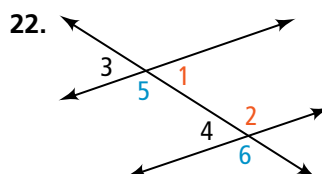
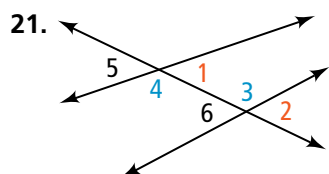
See Problem 2.

17. corresponding angles
18. alternate interior angles
19. same-side interior angles
20. alternate exterior angles



Are the angles labeled in the same color *alternate interior angles*, *same-side interior angles*, *corresponding angles*, or *alternate exterior angles*?

See Problem 3.



24. **Aviation** The photo shows an overhead view of airport runways. Are $\angle 1$ and $\angle 2$ *alternate interior angles*, *same-side interior angles*, *corresponding angles*, or *alternate exterior angles*?



B Apply

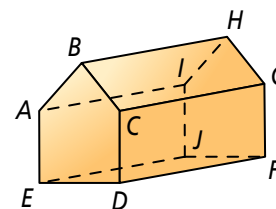
How many pairs of each type of angles do two lines and a transversal form?

- | | |
|-------------------------------|--------------------------|
| 25. alternate interior angles | 26. corresponding angles |
| 27. alternate exterior angles | 28. vertical angles |

29. **Recreation** You and a friend are driving go-karts on two different tracks. As you drive on a straight section heading east, your friend passes above you on a straight section heading south. Are these sections of the two tracks *parallel*, *skew*, or *neither*? Explain.

In Exercises 30–35, describe the statement as *true* or *false*. If false, explain. Assume that lines and planes that appear to be parallel are parallel.

- | | |
|---|---|
| 30. $\overleftrightarrow{CB} \parallel \overleftrightarrow{HG}$ | 31. $\overleftrightarrow{ED} \parallel \overleftrightarrow{HG}$ |
| 32. plane $AED \parallel$ plane FGH | 33. plane $ABH \parallel$ plane CDF |
| 34. \overleftrightarrow{AB} and \overleftrightarrow{HG} are skew lines. | 35. \overleftrightarrow{AE} and \overleftrightarrow{BC} are skew lines. |



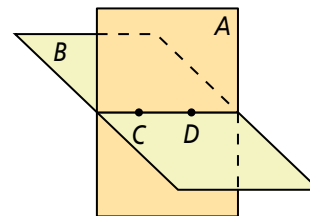
36. **Think About a Plan** A rectangular rug covers the floor in a living room. One of the walls in the same living room is painted blue. Are the rug and the blue wall parallel? Explain.
- Can you visualize the rug and the wall as geometric figures?
 - What must be true for these geometric figures to be parallel?

In Exercises 37–42, determine whether each statement is *always*, *sometimes*, or *never* true.

37. Two parallel lines are coplanar.
38. Two skew lines are coplanar.
39. Two planes that do not intersect are parallel.
40. Two lines that lie in parallel planes are parallel.
41. Two lines in intersecting planes are skew.
42. A line and a plane that do not intersect are skew.
43. a. **Writing** Describe the three ways in which two lines may be related.
b. Give examples from the real world to illustrate each of the relationships you described in part (a).
44. **Open-Ended** The letter Z illustrates alternate interior angles. Find at least two other letters that illustrate pairs of angles presented in this lesson. Draw the letters. Then mark and describe the angles.
45. a. **Reasoning** Suppose two parallel planes A and B are each intersected by a third plane C . Make a conjecture about the intersection of planes A and C and the intersection of planes B and C .
b. Find examples in your classroom to illustrate your conjecture in part (a).



Use the figure at the right for Exercises 46 and 47.



46. Do planes A and B have other lines in common that are parallel to \overleftrightarrow{CD} ? Explain.
47. **Visualization** Are there planes that intersect planes A and B in lines parallel to \overleftrightarrow{CD} ? Draw a sketch to support your answer.
48. **Draw a Diagram** A transversal r intersects lines ℓ and m . If ℓ and r form $\angle 1$ and $\angle 2$ and m and r form $\angle 3$ and $\angle 4$, sketch a diagram that meets the following conditions.
- $\angle 1 \cong \angle 2$
 - $\angle 3$ and $\angle 4$ are supplementary.
 - $\angle 3$ is an interior angle.
 - $\angle 2$ and $\angle 4$ lie on opposite sides of r .
 - $\angle 4$ is an exterior angle.

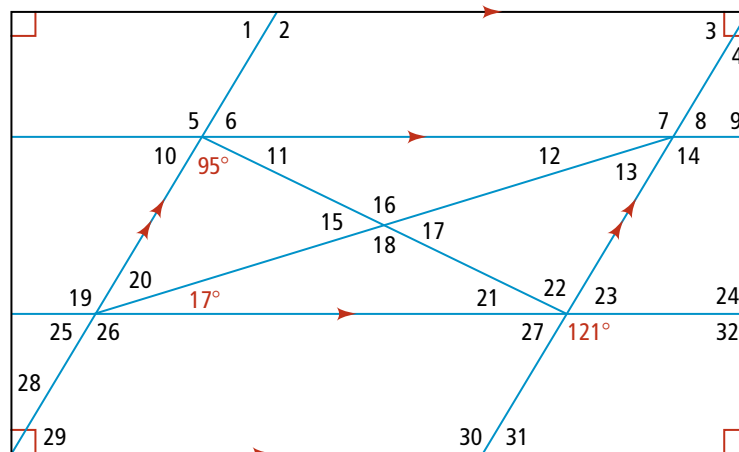


Apply What You've Learned



MP 1, MP 7

Look back at the information on page 139 about the plan for a city park. The blueprint of the park plan is shown again below.



The blueprint contains many examples of the types of angle pairs you learned about in Lesson 3-1, some formed by parallel lines and a transversal, and some formed by nonparallel lines and a transversal. Name all numbered angles in the blueprint that fit the given description.

- an angle that forms a pair of alternate interior angles with $\angle 2$
- an angle that forms a pair of corresponding angles with the angle that measures 121°
- an angle that forms a pair of same-side interior angles with the angle that measures 121°
- an angle that forms a pair of alternate interior angles with the angle that measures 95°